MATHEMATICAL MODELING OF COMPLEX HEAT TRANSFER

IN SLIT CHANNEL

S. P. Detkov and O. A. Khalevich UDC 536.3

A system of energy-transfer equations in layers of water vapor and a gray medium is given and numerically solved, taking account of radiation turbulent and molecular heat conduction.

1. The conditions of the problem are as follows. In a plane layer with surface temperatures $T_1 > T_2$ and reflection coefficients R_1 and R_2, there moves a medium selectively absorbing, radiating, and scattering energy. All the physical characteristics, including the albedo and the specific power of the chemical reactions, vary over the depth. Following the example of [1], the temperature field is regarded as one-dimensional. The problem is steady-state. The temperature field and energy fluxes in the medium are determined.

2. The energy balance of a volume element is written in dimensionless form

$$\frac{\partial}{\partial\tau}\left(N_e\frac{\partial\theta}{\partial\tau}\right)=(1-\omega)\,(\theta^4-\theta_*^4-g_*) \quad (1)$$

with the boundary conditions $\theta = \theta_1$ when $\tau = 0$ and $\theta = \theta_2$ when $\tau = \tau_0$, where

$$\tau=\int_0^y k\,dy';\ k=\alpha+\beta;\ \alpha=\sum_{*i}\alpha_{*i}+\sum_i\alpha_i p_i,$$

$$\theta^4=\eta_I/4n^2\alpha\sigma T_0^4;\ \theta_*^4=\eta_{ab}/4n^2\alpha\sigma T_0^4,\ \eta_I=4\alpha n^2\sigma T^4,$$

$$N_e=N\,(1+\mathrm{Pr}\,\varepsilon_\tau/\nu),\ N=k\lambda/4n^2\sigma T_0^3,\ \omega=\beta/k,$$

$$g_*=g/4n^2\alpha\sigma T_0^4,\ \theta=T/T_0.$$

3. The method of solution adopted is organically related to the form of Eq. (1) and is being tested for the first time in the formulated problem. It involves quasilinearization of Eq. (1):

$$\theta^4-\theta_*^4=(\theta-\theta_*)\,[(\theta+\theta_*)\,(\theta^2+\theta_*^2)]. \quad (2)$$

The value of θ is substituted into the square brackets in the zero approximation, and is found in the round brackets by solving Eq. (1) using the standard scheme of the finite-difference method. The value of θ_* is found from the equation

$$4\theta_*^4 n^2\sigma T_0^4\alpha=\eta_{ab}=\eta_I-\mathrm{div}\,\mathbf{q}_r. \quad (3)$$

This constitutes the first step in the successive approximation of the temperature field. Usually the function $\theta(\tau)$ converges after 8-12 steps.

The first feature of the method of solution, as is evident, is that the dimensionless density of the absorbed radiant flux θ_*^4 is isolated. This allows the calculation of the radiant heat transfer to be separated from the calculation of the turbulent and molecular heat conduction and facilitates the direct solution of Eq. (1). Any method may be used to calculate θ_*^4; it does not depend on Eq. (1). Another important feature of the method is that the dimensionless optical depths are determined on the basis of the mean Planck absorption coefficient α and the scattering coefficients, which depend on the spectra of the bodies.

All-Union Scientific-Research Institute of Energy and Nonferrous Materials, Sverdlovsk.
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4. The spectra of the bodies are taken into account in the calculations of the attenuation coefficient and the volume density of the absorbed flux θ_*^4. The general, more correct method is used to calculate the spectral quantities η_{ab}, which are then summed over the spectrum. Equation (3) is not affected by this, and is used to monitor the final results.

In the present work, another method is used to calculate θ_*^4, on the basis of the integral (over the spectrum) absorptive capacity of the medium for an incident flux with a black spectrum. From the value of a, the density of the hemispherical fluxes is found, and then

$$\operatorname{div} q_r \approx [(q_+ - q_-)_k - (q_+ - q_-)_{k-1}]/\Delta y.$$

Finally, η_{ab} is determined from Eq. (3).

5. Consider the calculation of the fluxes q_+ and q_- in a selective medium without scattering in a channel with gray walls. The wall radiation is assumed to be isotropic, and the reflection specular. Five "forward" fluxes from different sources and initial directions are shown in Fig. 1. In the channel they are reflected and, at a depth of τ_k, summed:

$$q_{+1} = (1 - R_1)\,\sigma T_1^4 \sum_{j=0}^{\infty} (R_1R_2)^j\,[1 - a(\tau_k + 2j\tau_0)],$$

$$q_{+2} = (1 - R_2)\,R_1\sigma T_2^4 \sum_{j=0}^{\infty} (R_1R_2)^j\,[1 - a(\tau_k + \langle 2j + 1\rangle\,\tau_0)],$$

$$q_{+3} = \sum_{i=1}^{k} \sigma T_i^4\,[a(\tau_k - \tau_{i-1}) - a(\tau_k - \tau_i)],$$

$$q_{+4} = \sum_{i=1}^{n} \sigma T_i^4 R_1 \sum_{j=0}^{\infty} (R_1R_2)^j\,[a(\tau_k + \tau_i + 2j\tau_0) - a(\tau_k + \tau_{i-1} + 2j\tau_0)],$$

$$q_{+5} = \sum_{i=1}^{n} R_1R_2\sigma T_i^4 \sum_{j=0}^{\infty} (R_1R_2)^j\,[a(\tau_k - \tau_{i-1} + 2\langle j + j\rangle\,\tau_0) - a(\tau_k - \tau_i + 2\langle j + 1\rangle\,\tau_0)],$$

where i − 1 and i are the boundaries of region i along the τ axis. The fluxes q_{-t} are calculated from analogous formulas. The partial fluxes are summed:

$$q_{+k} = \sum_{t=1}^{5} q_{+t},\quad q_{-k} = \sum_{t=1}^{5} q_{-t}.$$

Usually, the reflection coefficients are small and it is sufficient to take account of the first 4-6 terms of the series. Where this is not the case, the use of the following approximate formula, obtained on the basis of [2], is recommended

$$\sum_{j=1}^{\infty} (R_1R_2)^j\,a(\tau + 2j\tau_0) = \frac{R_1R_2}{1 - R_1R_-}\;\frac{a(\tau + 2\tau_0)}{1 - R_1R_2\sqrt{\dfrac{a(\tau + 6\tau_0) - a(\tau + 4\tau_0)}{a(\tau + 2\tau_0)}}},$$

where τ takes values from zero to $2\tau_0$ for the different fluxes. The error of the approximation for the calculated fluxes is usually considerably less than 1%. It is zero for a gray medium and isotropic reflection with τ = 0 and any value of τ_0.

6. Consider the transition to the limit of the formula in Sec. 5 for a gray medium. In this way, incidentally, isotropic scattering of the medium is taken into account. On the basis of the typical equation (isotropic reflection)

$$1 - a(\tau + 2j\tau_0) = 2E_3(\tau)\,2^{2j}E_3^{2j}(\tau_0)$$

with the notation $2E_n \equiv w_n$ and $q \equiv q/n^2\sigma T_0^4$, the following expressions are obtained:

$$q_{+1} = (1 - R_1)\,\theta_1^4 w_3(\tau)/z,\quad q_{+2} = R_1(1 - R_2)\,\theta_2^4 w_3(\tau)\,w_3(\tau_0)/z,$$

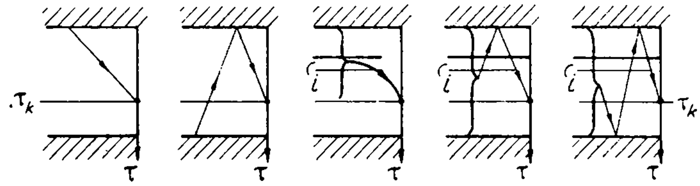

Fig. 1. Five component "forward" fluxes q_{+t} (at a depth of τ_k) from different primary sources and directions.

$$q_{+3} = \int_0^{\tau} B(\tau') w_2(|\tau - \tau'|)\, d\tau',$$

$$q_{+4} = R_1 w_3(\tau) \int_0^{\tau_0} B(\tau) w_2(\tau)\, d\tau / z,$$

$$q_{+5} = R_1 R_2 w_3(\tau) w_3(\tau_0) \int_0^{\tau_0} B(\tau) w_2(\tau_0 - \tau)\, d\tau / z,$$

where $z = 1 - R_1 R_2 W_3^2(\tau_0)$. Here and below, isotropic scattering in the medium is taken into account with $B = (1 - \omega)\theta^4 + \omega\theta_*^4$ and not $B = \theta^4$ in the formula of Sec. 5.

Summing the fluxes gives

$$(q_+)_k \equiv q_+(\tau_k) = q_{e1} w_3(\tau_k) + \int_0^{\tau_k} B(\tau) w_2(\tau_k - \tau)\, d\tau,$$

$$(q_-)_k \equiv q_-(\tau_k) = q_{e2} w_3(\tau_0 - \tau_k) + \int_{\tau_k}^{\tau_0} B(\tau) w_2(\tau - \tau_k)\, d\tau,$$

$$q_{e1} = \frac{b_1 + b_2 d_1}{1 - d_1 d_2}, \quad q_{e2} = \frac{b_2 + b_1 d_2}{1 - d_1 d_2}, \quad d_1 = R_1 w_3(\tau_0), \quad d_2 = R_2 w_3(\tau_0),$$

$$b_1 = (1 - R_1)\theta_1^4 + R_1 \int_0^{\tau_0} B w_2(\tau)\, d\tau, \quad b_2 = (1 - R_2)\theta_2^4 + R_2 \int_0^{\tau_0} B w_2(\tau_0 - \tau) d\tau.$$

The equation for θ_*^4 is well known [3]:

$$4\theta_*^4 = q_{e1} w_2(\tau) + q_{e2} w_2(\tau_0 - \tau) + \int_0^{\tau_0} B(\tau') w_1(|\tau - \tau'|)\, d\tau'.$$

The three integral equations written for q_{e1}, q_{e2}, and θ_*^4, and also Eq. (1), are solved simultaneously at each step of the iteration for the temperature field in the medium. The results of solution are given in Sec. 9.

The equations of this section are easily transformed for a spectral interval, by averaging the absorption and scattering coefficients in it. The sum of the solutions of the equations over the whole spectral interval lead to the value of θ_*^4 required for the solution of Eq. (1). In this case, the spectra of all the bodies are taken into account. A method of taking into account the scattering anisotropy is described in Sec. 10.

7. In the case of a medium including only one absorbing gas component, it is expedient to use, in parallel with the τ axis, the axis of the depth x, m•atm, in terms of the partial pressure p:

$$x = \int_y p\, dy', \quad x_0 = \int_0^{y_0} p\, dy'.$$

In the present work, numerical calculations are carried out for water vapor, using a new formula for the absorptive capacity with respect to an incident flux with a blackbody spectrum, published in [4]. In the given problem, the primary sources of flux are not only surfaces but also regions, the elementary layers of the medium. The absorptive capacity of the gas for the flux emitted by the i-th region is described on the basis of reciprocal relations using the same formula with a general notation for the temperature of the region (the flux source) T_i regardless of the form of the region.

The directed absorptive capacity is integrated approximately over the solid angle. For a plane layer, according to [5], the following relation holds

$$a(x) = (\overline{T}/T_i)^m \, \varepsilon_* [T_i, \; x(\overline{T}/T_i)^u],$$

where

$$\varepsilon_* = 0.0628\,\varepsilon(8.8\,x') + 0.4444\,\varepsilon(2x') + 0.4928\,\varepsilon(1.125\,x'),$$

ε is the directed emissivity for thicknesses $x' = x(\overline{T}/T_i)^u$, multiplied by 8.8, 2, and 1.125. The formula for ε and the power-law indices m and u are given in [4]. The quantity $\overline{T}$ is the temperature of the absorbing medium averaged according to the simplest formula

$$\overline{T} = \frac{1}{x}\int_0^x T(x_1)\,dx_1,$$

where T is the local temperature at a depth $0 \leqslant x_1 \leqslant x$. The value of x is composite if multiple reflection of the flux occurs. The basis for this formula is given in [6] and elsewhere.

The new formula for the absorptive capacity, in contrast to the well-known Hottel formula, gives a physically acceptable value of $a(x)$ for any argument x. This is important because the series in Sec. 5 include a with unbounded growth of the optical thickness.

8. The heat-transfer coefficients are taken on the assumption of completely developed turbulent flow along the channel walls. According to a three-layer scheme

$$0 \leqslant y^+ \leqslant 5,\ \varepsilon_T/\nu = 0,\ 5 < y^+ \leqslant 30,\ \varepsilon_T/\nu = 0.2y^+ - 1,$$
$$30 < y^+ \leqslant y_0^+/2,\ \varepsilon_T/\nu = 0.1y^+(1.82 - 2y^+/y_0).$$

The third formula for the flow core is written on the basis of continuity of the functions u(y), $\varepsilon_T/\nu = f(y)$, and therefore differs from the formula published in [7]. This is insignificant. What is much more important is the value of ε_T/ν in the boundary layer, where the pulsations of the medium are damped. A check of the version with a single formula for ε_T/ν given in [1] showed that the heat fluxes obtained for all the transfer mechanisms are too high by ~3%. The difference is explained almost exclusively in that according to the single formula $\varepsilon_T/\nu > 0$ in the boundary layer.

9. Numerical results for a system of gray bodies are given in Tables 1 and 2 and Fig. 2 for $y_0^+ = 400$, $V^+ = 19.76$, Re = 3953, Pr = 1. The well-known data given in [3] and elsewhere are nothing like as complete as those of the present calculations. They were reproduced in the present program as a control, and the discrepancy did not exceed a few units in the fourth significant figure.

For water vapor mixed with nitrogen, the algorithm has as yet been checked only for the particular results published in the literature. In Fig. 3, the temperature field with the left-hand side of Eq. (1) set equal to zero is shown, taking account only of the radiation mechanism. Comparison with the curves of [8] reveals slight discrepancies. The present results are more accurate, since the radiation of the gas is determined from its absorptive capacity, the temperature field is taken into account for the absorption path, more accurate formulas are used for the absorptive capacity and emissivity of the gas and the integral over the solid angle, and the broadening of the spectral lines is taken into account.

In the case of cold walls and a given linear temperature field, the very particular problem first solved by Kavaderov is obtained. The solutions of this problem are reviewed in [9]. A summary of some of the data is given in Table 3; for comparison, data obtained on a simpler basis in [10] are also given.

TABLE 1. Dimensionless Density ($q \equiv q/n^2\sigma T_0^4$) of Hemispherical Fluxes (convective, radiant, total, and effective at the layer boundaries 1 and 2)*

N	$\tau = 0$				$\tau = \tau_0$				$q_2 - q_1$	No. of iterations
	q_{c1}	q_{r1}	q_1	q_{e1}	q_{c2}	q_{r2}	q_2	q_{e2}		
0	0,0000	−1,9909	−1,9909	1,4977	0,0000	1,2719	1,2719	1,2720	3,2628	23
0,001	−0,1271	−1,7806	−1,9077	1,4452	0,2173	1,1365	1,3538	1,1366	3,2615	24
0,01	−0,5889	−1,0024	−1,5913	1,2506	0,9059	0,7668	1,6723	0,7666	3,2637	21
0,1	0,4465	0,4412	0,8876	0,8897	3,9508	0,2018	4,1526	0,2019	3,2649	10
1	24,2589	0,6131	24,8720	0,8467	27,9968	0,1416	28,1384	0,1417	3,2664	6
					Accurate divergence of fluxes q_2 and q_1				3,2620	

*The ratio ε/ν is calculated by a three-layer scheme with $y_0^+ = 400$, Re = 3953, Pr = 1, and n = 1.

TABLE 2. Dimensionless Thermal-Flux Densities at the Layer Boundaries*

N	$\tau = 0$				$\tau = \tau_0$				$q_2 - q_1$	No. of iterations
	q_{c1}	q_{r1}	q_1	q_{e1}	q_{c2}	q_{r2}	q_2	q_{e2}		
0	0	0,0435	0,0435	0,9891	0	0,4437	0,4437	0,4437	0,4002	15
0,001	−0,0437	0,1228	0,0791	0,9693	0,0730	0,4065	0,4794	0,4066	0,4003	18
0,01	−0,0070	0,3488	0,3418	0,9128	0,4402	0,3023	0,7425	0,3024	0,4006	18
0,1	2,3305	0,4138	2,7443	0,8965	2,8775	0,2721	3,1496	0,2722	0,4052	12
1	26,3020	0,4029	26,7048	0,8993	26,8667	0,2797	27,1463	0,2798	0,4415	7
					Accurate divergence of fluxes q_1 and q_2				0,3998	

*The conditions of Table 1 are retained, except that the albedo is assumed to be constant: $\omega = 0.9$. The temperature field is shown in Fig. 2b.

TABLE 3. Hemispherical Fluxes Emitted by a Water-Vapor Layer with Cold Black Surfaces*

x_0, m·atm	q_+, kW/m²				q_-, kW/m²			
	1	2	3	4	1	2	3	4
0,001	0,458	0,446	0,447	0,824	0,460	0,446	0,447	0,865
0,01	4,10	4,21	4,24	6,84	4,23	4,23	4,26	7,79
0,05	14,6	16,75	17,20	21,61	16,40	17,66	18,00	29,65
0,1	23,2	26,4	27,5	30,8	27,4	30,2	31,0	48,1
0,5	51,4	47,8	50,7	52,9	80,0	82,5	85,6	114,7
1	—	—	57,3	62,4	—	—	119,5	150,9
10	—	—	75,3	87,2	—	—	254,9	281,2

*The temperature field in the layer is linear: $T = 1000(2 - x/x_0)$, P = 1 atm, p = 0.1 atm; $R_1 = R_2 = \theta_1 = \theta_2 = \omega = 0$. The column numbers correspond to the following conditions; 1) calculation according to [10] (the approximation $m = u = 0$, $P_* = 1$ atm); 2) present calculation with the assumptions of [10]; 3) present calculation with $m = u = 0$, $P_* = P + 5p\sqrt{273/T}$; 4) present calculation with the values of m and u given in the text and effective pressure $P_* = P + 5p\sqrt{273/T}$.

10. Taking account of scattering anisotropy in a plane layer is technically complex, but does not involve any fundamental difficulties. The scattering index must be written as a series in Legendre polynomials. The integral equation for θ_*^4 is complicated and, in addi-

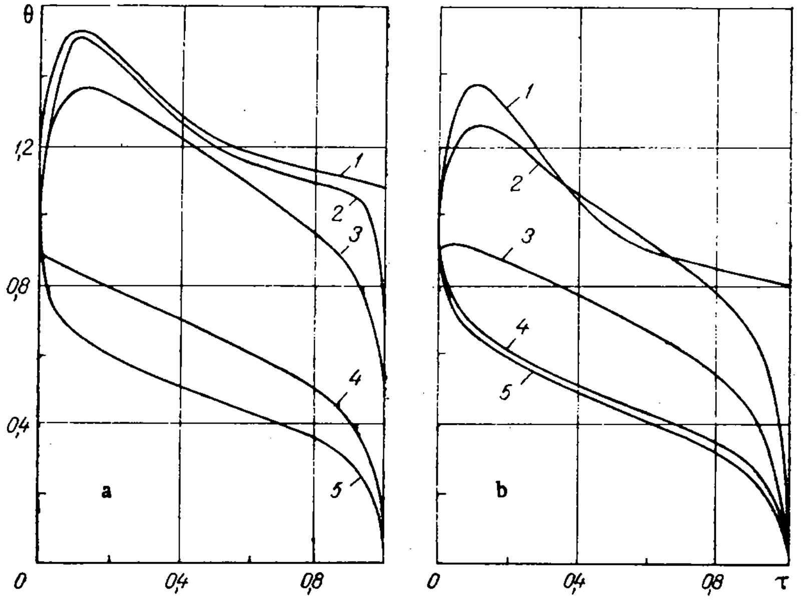

Fig. 2. Temperature field in a gray medium (a) under the conditions: $\theta_1 = 1$; $\theta_2 = 0.1$; $\tau_0 = 1$; $R_1 = 0.2$; $R_2 = 0.5$; Pr = 1; $\omega = 0.1 + 0.6\ (\tau/\tau_0)[1 -(\tau/\tau_0)]$ (a); $g_* = 100(\tau/\tau_0)\exp(-10\ \tau/\tau_0)$. The curves are numbered in order of increasing N; 0, 0.001, 0.01, 0.1, 1. The flux densities at the layer boundaries are given in Table 1. The coefficients ε_T/ν are taken from a three-layer scheme with separation of the boundary layer; $y_0^+ = 400$; Re = 3953; $v^+ = 19.76$; b) the temperature field in the case of constant albedo: $\omega = 0.9$; all other conditions are the same.

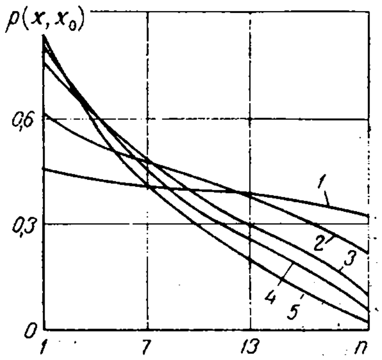

Fig. 3. Temperature field in an adiabatic layer of water vapor with $T_1 = 2000°K$, $T_2 = 1000°K$, $g_* = \omega = N_e = 0$; P = 1 atm; p = 0.1 atm; $P_* = P + 5p\sqrt{273/T}$. The numbering of the curves corresponds to the thickness x, m·atm: 0.01, 0.1, 1, 10, 100; n is the number of the region for uniform division of the segment x_0, m·atm. The ordinate is given in the traditional form($p = (T^4 - T_2^4)/(T_1^4 - T_2^4)$).

tion, there appear new integral equations in accordance with the number of terms of the series. All the integral equations, including those for q_{e_1} and q_{e_2}, must be solved simultaneously at each step of the iterative approximation of the temperature field. Equation (1) and the method of its solution remain unchanged.

NOTATION

a, dimensionless absorptive capacity of the medium for a flux with a blackbody spectrum; c_p, specific heat, J/kg·°K; g, specific power of chemical reactions, W/m³ ; $k = \alpha + \beta$, attenuation coefficient, m^{-1}; n, refractive index; q, density of hemispherical flux, W/m² or dimensionless q_{+k}, q_{-k}, "forward" and "back" fluxes at depth τ_k; q_c, q_r, density of convective and radiant heat fluxes; u, local velocity of medium along the surface, m/sec; u_*, dynamic velocity, m/sec; v^+, velocity averaged over the channel cross section, dimensionless; y, coordinate from surface 1 along the normal into the depth of the medium, m; Pr, Prandtl number; Re, Reynolds number; R, surface-reflection coefficient; N, conductive-radiational parameter, dimensionless; N_e, its effective value, taking into account heat conduction and turbulization of the medium; T, T_1, T_2, T_0, temperature of the medium, surface temperatures, standard temperature (T_0 = 1000°K); α_{*i}, α_i, mean Planck absorption coefficients of the suspended-phase and gas components; β, scattering coefficient averaged over the spectrum, m^{-1}; ε_T, turbulent-heat-transfer coefficient, m^2/sec; ε, ε_*, directed emissivity and emissivity integrated over the solid angle for the layer; λ, thermal conductivity, W/m·°K; ν, kinematic viscosity, m^2/sec; τ, optical depth in terms of the attenuation coefficient, dimensionless; x, optical depth in terms of the partial pressure of the gas component, m·atm; τ_0, x_0, optical thickness; p, P, P_*, partial, total, and effective pressure, atm; η_I, η_{ab}, volume density of the intrinsic and absorbed fluxes, W/m^3. Indices: 1, 2, channel surfaces at τ = 0, τ_0; e, effective value; i, region number; j, reflection number. The dimensionless quantities are defined as follows

$$\theta = T/T_0,\ \theta_1 = T_1/T_0,\ \theta_2 = T_2/T_0,\ \tau_0 = ky_0,$$

$$\omega = \beta/k,\ y^+ = yu_*/\nu,\ u^+ = u/u_*,\ \Pr = c_p\rho\nu/\lambda,$$

$$\theta_*^4 = \eta_{ab}/(4n^2\alpha\sigma T_0^4),\ N = k\lambda/(4n^2\sigma T_0^3),$$

$$N_e = N(1 + \Pr\varepsilon_T/\nu),\ w_n(\tau) \equiv 2E_n(\tau) = 2\int_0^1 \exp\left(-\frac{\tau}{\mu}\right)\mu^{n-2}\,d\mu,$$

$$q_c(\tau) \equiv \frac{q_c}{n^2\sigma T_0^4} = -4N_e\frac{\partial\theta}{\partial\tau},$$

$$q_r(\tau) \equiv \frac{q_r}{n^2\sigma T_0^4} = \theta_1^4 w_3(\tau) - \theta_2^4 w_3(\tau_0 - \tau) +$$

$$+\int_0^\tau B(\tau')\,w_2(\tau - \tau')\,d\tau' - \int_\tau^{\tau_0} B(\tau')\,w_2(\tau' - \tau)\,d\tau'.$$

LITERATURE CITED

1. A. Balakrishnan and D. K. Edwards, Report of the Proceedings of the Fifth International Heat Transfer Conference, Vol. 1, Tokyo (1974), p. 93.
2. S. P. Detkov, Inzh.-Fiz. Zh., 18, No. 5, 5 (1970).
3. M. N. Ozisik, Radiative Heat Transfer and Internations with Conduction and Convection, Wiley, New York (1973).
4. S. P. Detkov and O. A. Khalevich, Inzh.-Fiz. Zh., 35, No. 3, 486 (1978).
5. S. P. Detkov, Inzh.-Fiz. Zh., 21, No. 2, 205 (1971).
6. S. P. Detkov, Inzh.-Fiz. Zh., 20, No. 5, 802 (1971).
7. W. M. Kays, Convective Heat and Mass Transfer, McGraw-Hill (1966).
8. S. P. Detkov and A. V. Vinogradov, Izv. Akad. Nauk SSSR, Energ. Transp., No. 3, 139 (1969).
9. S. P. Detkov, Teplofiz. Vys. Temp., 8, No. 4, 840 (1970).
10. A. S. Nevskii and L. A. Chukanov, Teplofiz. Vys. Temp., 3, No. 4,124 (1965).